RBF-FD BASED DYNAMIC THERMAL RATING OF OVERHEAD POWER LINES

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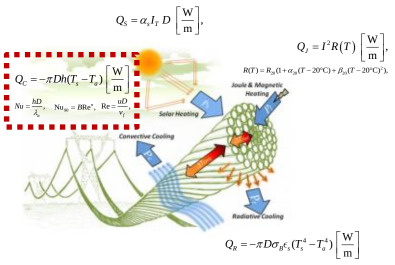
SCOPE

Cooling of conductor due to the natural convection

The most important cooling mechanism of a conductor, i.e. convection, is, in all standard Dynamic Thermal Rating (DTR) models, taken into account in terms of empirical relation that are mostly based on data collected by Morgan in 1975.



$$Q_S = \alpha_s I_T D \left[\frac{\text{W}}{\text{m}}\right],$$

$$Q_J = I^2 R(T) \left[\frac{\text{W}}{\text{m}}\right],$$

$$R(T) = R_{20}(1+\alpha_{20}(T-20°\text{C})+\beta_{20}(T-20°\text{C})^2),$$

$$Q_C = -\pi D h (T_s - T_a) \left[\frac{\text{W}}{\text{m}}\right]$$

$$Nu = \frac{hD}{\lambda_a}, \quad \text{Nu}_{90} = B\text{Re}^n, \quad \text{Re} = \frac{uD}{v_f},$$

$$Q_R = -\pi D \sigma_B \epsilon_s (T_s^4 - T_a^4) \left[\frac{\text{W}}{\text{m}}\right]$$

Physical model

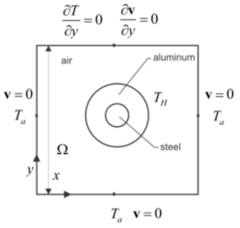
The domain of the problem is a cross-section of a power line that is further separated into a steel core and aluminum conductor, and surrounding air

$$\rho^a \frac{\partial \mathbf{v}}{\partial t} + \rho^a \nabla \cdot (\mathbf{v}\mathbf{v}) = -\nabla P + \nabla \cdot \left(\mu^a \nabla \mathbf{v}\right) + \mathbf{b}$$

$$\nabla \cdot \mathbf{v} = 0$$

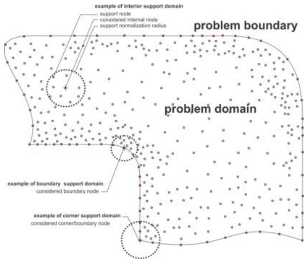
$$\rho^a c_p^a \frac{\partial T}{\partial t} + \rho^a c_p^a \nabla \cdot (T\mathbf{v}) = \nabla \cdot \left(\lambda^a \nabla T\right)$$

$$\mathbf{b} = \rho_{ref}\left[1 - \beta_T (T - T_{\text{ref}})\right]\mathbf{g}$$



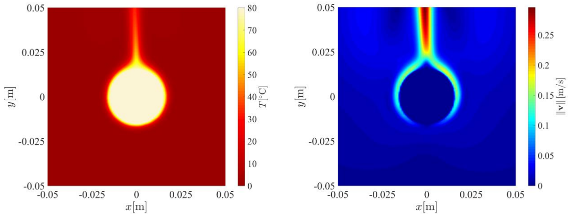
Numerical solution

The solution procedure is divided in two time loops, namely the air and the power line loop. The involved Partial Differential Equations are discretized with RBF-FD method.



Results

The results of the simulation are presented in terms of temperature and velocity magnitude contour plots, convergence analyses, and comparison of convective heat losses of simulated results to IEC, IEEE and CIGRE standards.



NUMERICAL MODEL VS. CIGRE/IEEE

Numerical solution of temperature and velocity fields around conductor

- Steady state achieved in order of 1 s
- Temporal discretization in order of 1 ms

CIGRE and IEEE assume empirical relation between temperature and cooling rate

In numerical model we solve thermo-fluid problem and compute cooling rate without any parameters.

Computation of heat generation and transport within the power line

- Steady state achieved in order of 10 min
- Temporal discretization in order of 10 s

CIGRE in IEEE assume homogenous conductor and assess the skin and core temperature from closed form solution of simplified problem.

In numerical model we compute 2D simulation of heat transfer and generation within the power line.

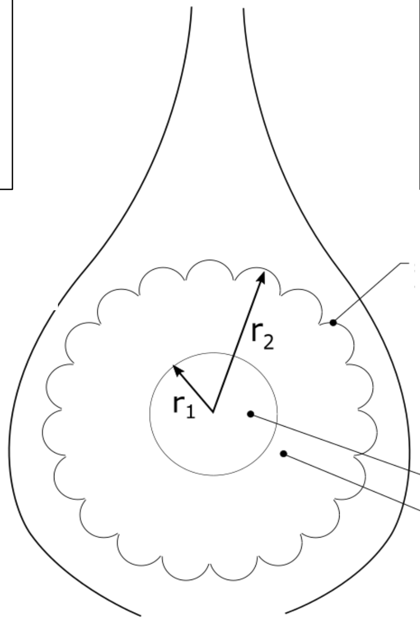
Natural convection:
Thermo-fluid problem

$$\rho^a \frac{\partial \mathbf{v}}{\partial t} + \rho^a \nabla \cdot (\mathbf{v}\mathbf{v}) = -\nabla P + \nabla \cdot \left(\mu^a \nabla \mathbf{v}\right) + \mathbf{b}$$

$$\nabla \cdot \mathbf{v} = 0$$

$$\rho^a c_p^a \frac{\partial T^a}{\partial t} + \rho^a c_p^a \nabla \cdot (T^a \mathbf{v}) = \nabla \cdot \left(\lambda^a \nabla T^a\right)$$

$$\mathbf{b} = \rho_{ref} \left[1 - \beta_T (T^a - T_{\mathrm{ref}})\right] \mathbf{g}$$



Radiation
Boundary condition

$$q_R = \sigma_B \epsilon_s \left(T_{al}^4(r_2) - T_a^4\right) \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

$$q_S = \frac{\alpha_s I_T}{\pi} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right],$$

Steel part – heat transport

$$T^{al}(r_1) = T^{st}(r_1)$$

Aluminium part – heat transport and heat generation

$$\rho^{st} \frac{\partial T^{st}}{\partial t} = \lambda^{st} \nabla^2 T^{st}$$

$$c_p^{al} \rho^{al} \frac{\partial T^{al}}{\partial t} = \lambda^{al} \nabla^2 T^{al} + q_J$$

$$q_j = \frac{I^2 R\left(T^{al}\right)}{S^{al}} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^3}\right],$$

$$\lambda^{al} \left.\frac{\partial T^{al}}{\partial \mathbf{n}}\right|_{r_1} = \lambda^{st} \left.\frac{\partial T^{st}}{\partial \mathbf{n}}\right|_{r_1}$$

$$\lambda^{al} \left.\frac{\partial T^{al}}{\partial \mathbf{n}}\right|_{r2} - \lambda^a \left.\frac{\partial T^a}{\partial \mathbf{n}}\right|_{r_2} = q_s + q_r$$

$$T^{al}(r_2) = T^a(r_2)$$

SOLUTION PROCEDURE

Main simulation loop

$$c_p^{st}\rho^{st}\frac{\partial T^{st}}{\partial t}=\lambda^{st}\nabla^2 T^{st}$$

Implicit solution of heat transport in steel part

$$c_p^{al}\rho^{al}\frac{\partial T^{al}}{\partial t}=\lambda^{al}\nabla^2 T^{al}+\frac{I^2R(T)}{S^{al}}$$

Implicit solution of heat transport and generation in aluminum part

Internal loop – Thermo fluid problem

$$\mathbf{v}^i=\mathbf{v}_1+\Delta t_a\left(\frac{1}{\rho^a}\nabla\cdot\left(\mu^a\nabla\mathbf{v}\right)-\nabla\cdot(\mathbf{vv})+\frac{\mathbf{b}}{\rho_a}\right)$$

Explicit solution of intermediate velocity

$$\nabla^2 p^c=\frac{\rho}{\Delta t_{air}}\nabla\cdot\mathbf{v}^i$$

$$\frac{\partial P}{\partial n}=\left(\nabla\cdot\left(\mu^a\nabla\mathbf{v}\right)+\mathbf{b}\right)\hat{\mathbf{n}}$$

$$\int_\Omega p\,d\Omega=0$$

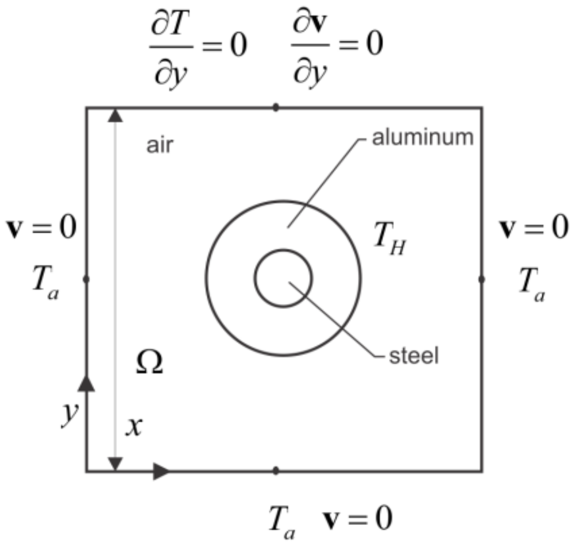
Pressure correction Poisson equation with regularization

$$\mathbf{v}_2=\mathbf{v}_1-\frac{\Delta t_a}{\rho}\nabla p^c$$

Velocity correction

$$T_2=T_1+\Delta t_a\left(\frac{1}{\rho^a c_p^a}\nabla\cdot\left(\lambda^a\nabla T\right)-\nabla\cdot(T\mathbf{v}_1)\right)$$

Explicit time advance of heat transport



RBF-FD DISCRETIZATION

Problem

$$Lu = a \qquad L = \nabla, \nabla^2, \frac{\partial}{\partial x^i}, \ldots$$

Approximate solution over a local support domain

$$\hat{u} = \sum_{i=1}^{m} \alpha_i b_i \dashrightarrow \hat{u} = \mathbf{b}^{\mathrm{T}} \boldsymbol{\alpha}$$

Approximation coefficients

Basis functions (MQs, monomials, Gaussians, …)

WLS

$$r^2 = \sum_{j}^{n} W_j \left(u_j - \hat{u}_j\right)^2$$

Moore Penrose pseudo inverse

$$\boldsymbol{\alpha} = \left(\mathbf{W}^{0.5}\mathbf{B}\right)^{-1}\left(\mathbf{W}^{0.5}\mathbf{u}\right)$$

B – matrix of dimension (*m* x *n*)
W – diagonal weight matrix

SVD / QR decomposition
/ NE - Cholesky decomposition

Special case Interpolation

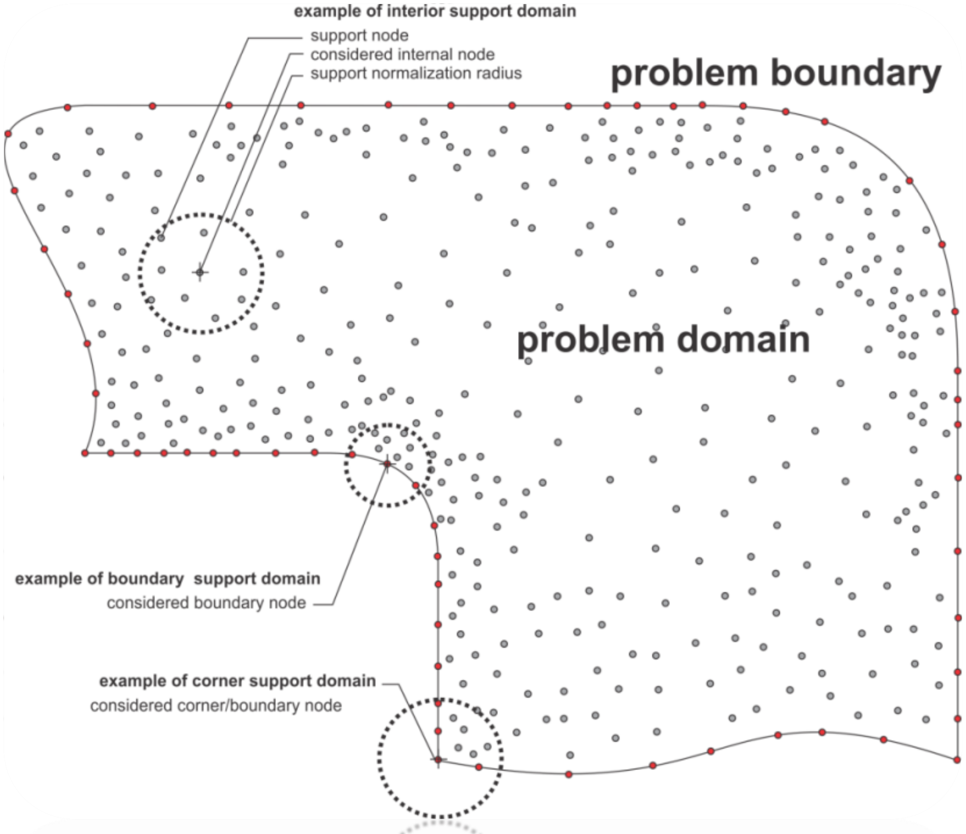
$$\boldsymbol{\alpha} = \mathbf{B}\mathbf{u}$$

Shape functions

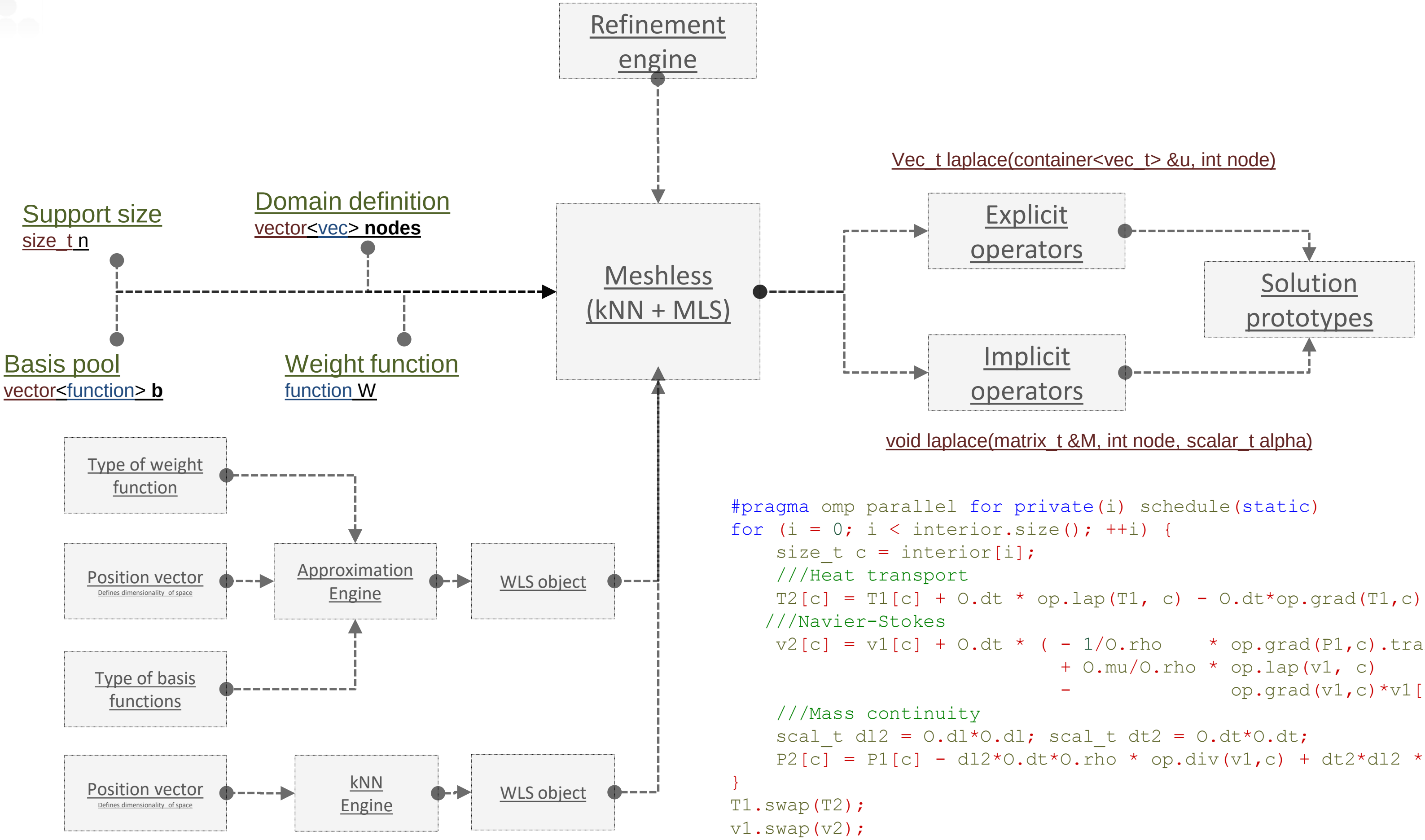
$$\hat{u} = \mathbf{b}^{\mathrm{T}}\left(\mathbf{W}^{0.5}\mathbf{B}\right)^{+}\mathbf{W}^{0.5}\mathbf{u} = \boldsymbol{\Phi}\mathbf{u}$$

Diff. operation

$$L\hat{u} = \underbrace{L\left(\mathbf{b}^{\mathrm{T}}\left(\mathbf{W}^{0.5}\mathbf{B}\right)^{+}\mathbf{W}^{0.5}\right)}_{\boldsymbol{\Phi}}\mathbf{u} = \sum_{j}^{n}\left(L\Phi\right)_j u_j$$



IMPLEMENTATION



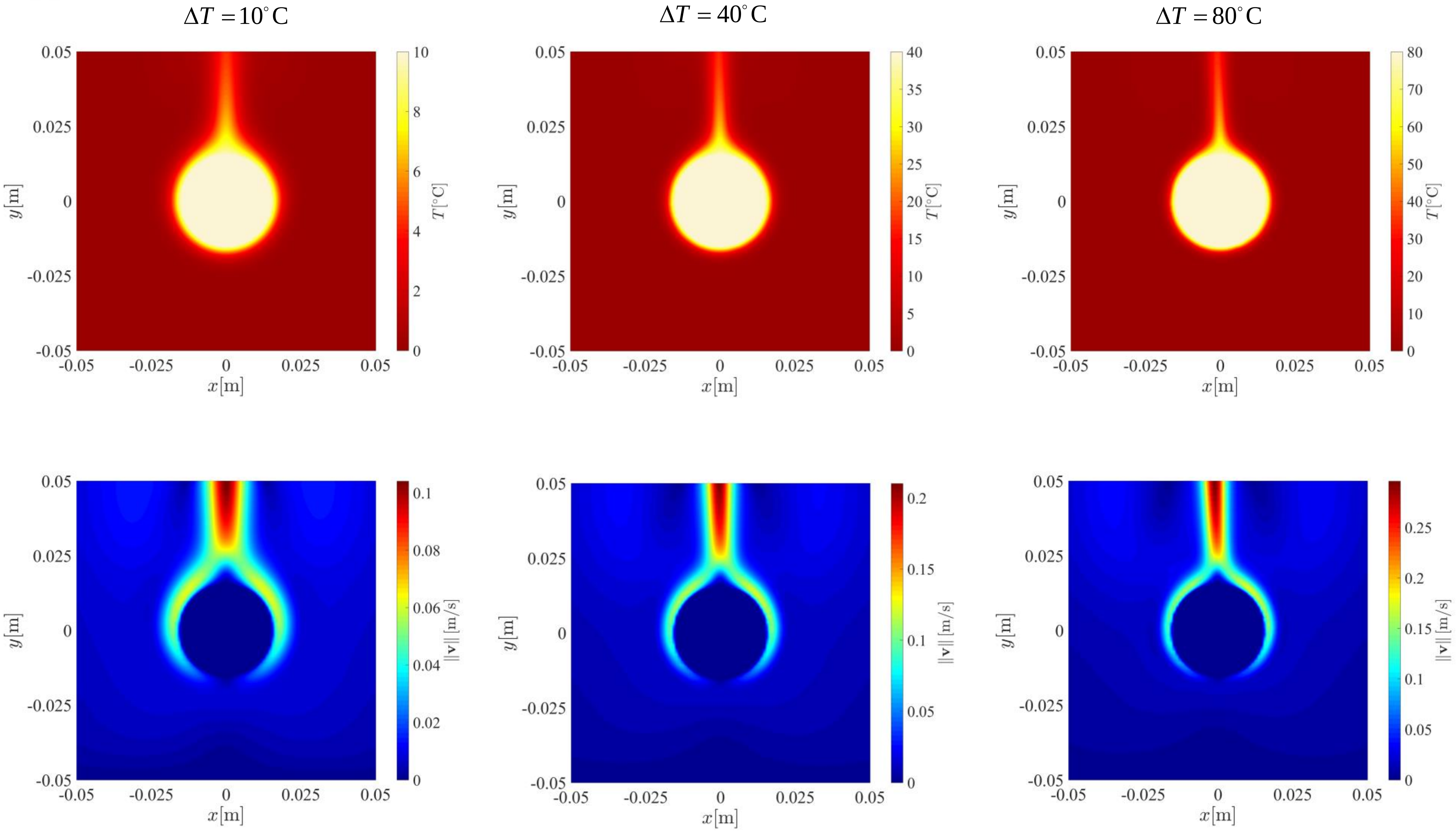
```
#pragma omp parallel for private(i) schedule(static)
for (i = 0; i < interior.size(); ++i) {
    size_t c = interior[i];
    ///Heat transport
    T2[c] = T1[c] + O.dt * op.lap(T1, c) - O.dt*op.grad(T1,c)*v1[c];
   ///Navier-Stokes
    v2[c] = v1[c] + O.dt * ( - 1/O.rho    * op.grad(P1,c).transpose()
                             + O.mu/O.rho * op.lap(v1, c)
                             -              op.grad(v1,c)*v1[c]);
    ///Mass continuity
    scal_t dl2 = O.dl*O.dl; scal_t dt2 = O.dt*O.dt;
    P2[c] = P1[c] - dl2*O.dt*O.rho * op.div(v1,c) + dt2*dl2 * op.lap(P1,c);
}
T1.swap(T2);
v1.swap(v2);
P1.swap(P2);
```

open source meshless project

Medusa: Coordinate Free Mehless Method implementation (MM)

https://gitlab.com/e62Lab/medusa | http://e6.ijs.si/medusa/

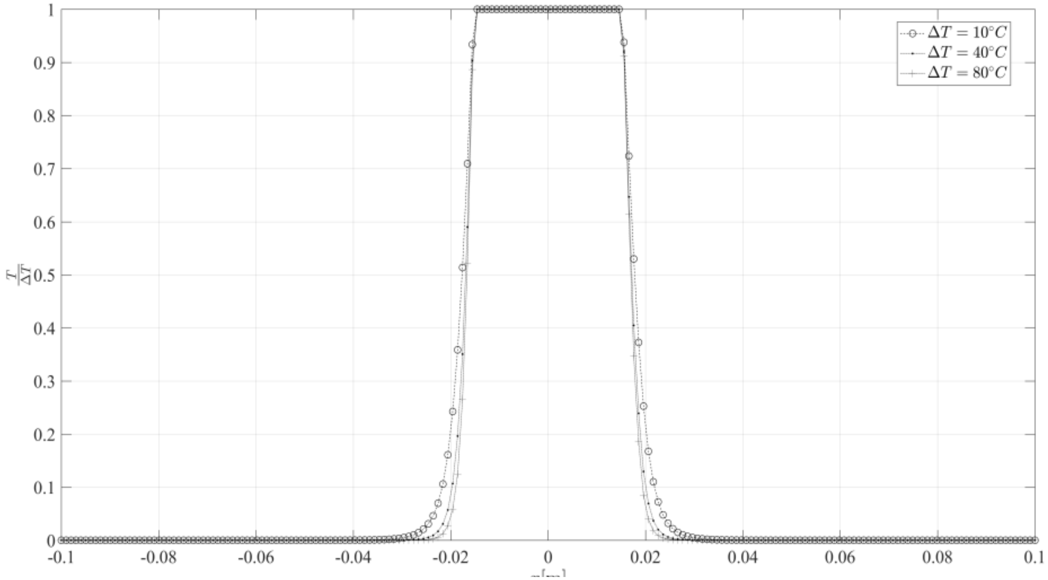
Simulation of natural convection from Al490Fe65 conductor


$\Delta T = 10^\circ\mathrm{C}$

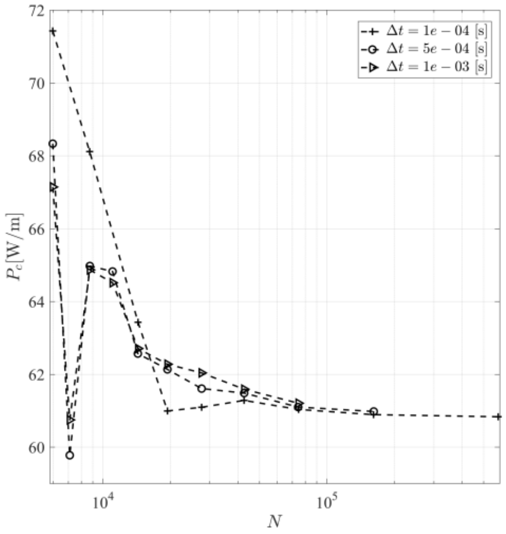
$\Delta T = 40^\circ\mathrm{C}$

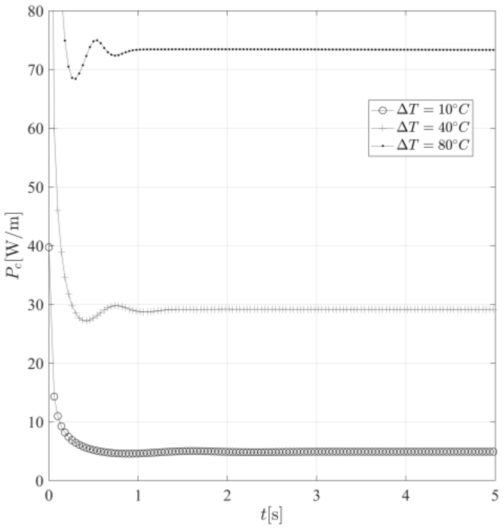
$\Delta T = 80^\circ\mathrm{C}$

Simulation of natural convection from Al490Fe65 conductor



Steady state temperature profile of air at different skin-ambient temperature differences.

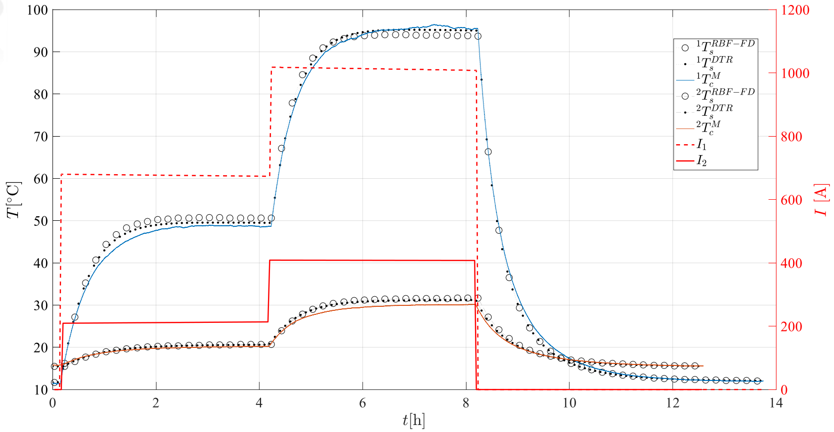


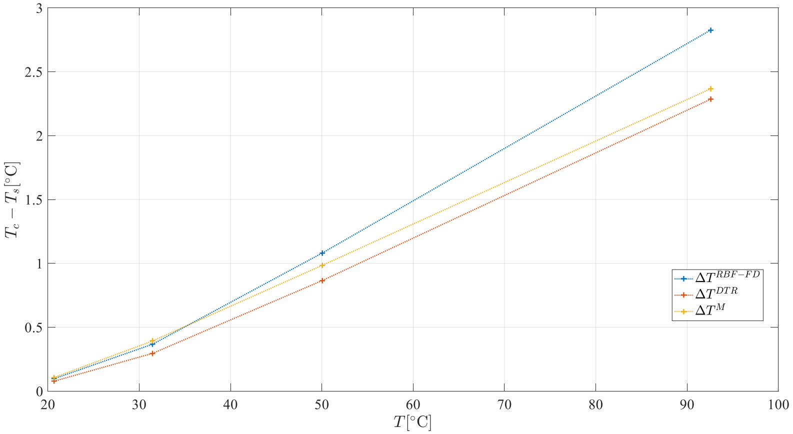


Convergence plot (left) and time development at different skin-ambient temperature differences

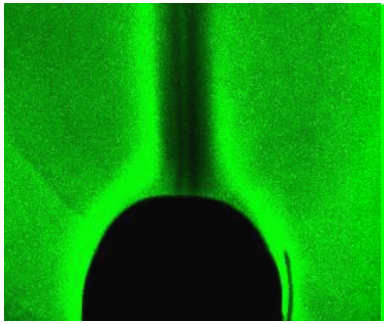
VALIDATION OF NUMERICAL SIMULATION

RBF-FD – numerical simulation, M - measurement

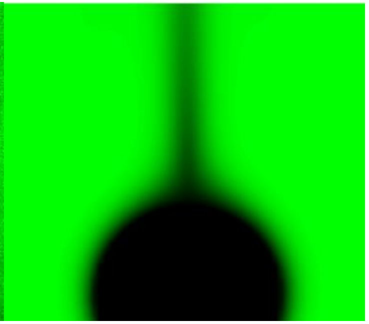




Experimental setup for measurements of conductor temperature



Result of Schlieren photography



Simulated temperature

Experimental setup for Schlieren photography.

CONCLUSIONS

Improving DTR algorithms

We prepared a numerical simulation of heat transport within the overhead line and thermo-fluid transport in surrounding air with the ultimate goal to further improve treatment of the most important cooling mechanism of overhead power line, i.e. convection.

The physical model is solved by an in-house implementation of RBF-FD method within the **Medusa** open source meshless project.

Model is validated by comparing simulation results, experimental data and IEEE and CIGRE standards.

Future work

Implement simulation of convective cooling in forced convection regime.

Prepare simulated relations for Nusselt number with respect to the wind velocity and geometry of the power line.

THANK YOU FOR YOUR ATTENTION