Refined RBF-FD Solution of Linear Elasticity Problem

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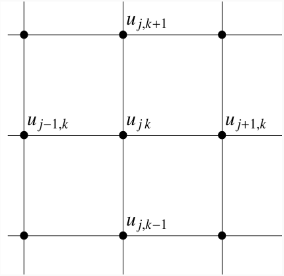
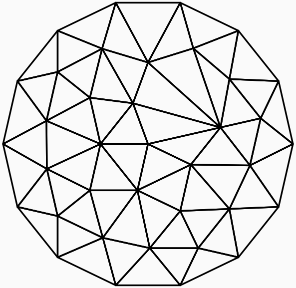
- Classical approaches:
  Finite Difference Method, Finite Element Method





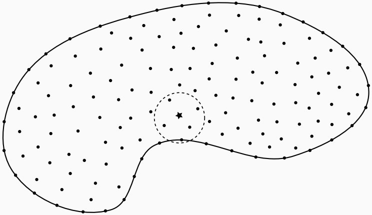
- Problems: inflexible geometry, mesh generation
- Response: mesh-free methods (EFG, MLPG, FPM)

Strong form meshless methods

JSI

Domain discretization:

- Points x_i on the boundary and in the interior
- Point neighborhoods $N(x_i)$



Classical Finite Differences:

$$u''(x_i) \approx \frac{1}{h^2}u(x_{i-1}) - \frac{2}{h^2}u(x_i) + \frac{1}{h^2}u(x_{i+1})$$

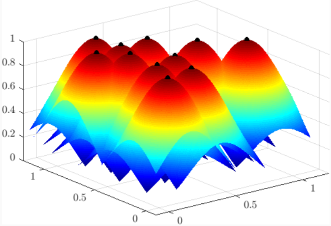
Generalized Finite Differences:

$$(\mathcal{L}u)(x_i) \approx \sum_{x_j \in N(x_i)} w_j^i u(x_j)$$

+ exactness for a certain set of functions (e.g. monomials)

Exactness is imposed for Radial Basis Functions

- Given nodes $X = \{x_1, \ldots, x_n\}$ and a radial function $\varphi = \varphi(r)$
- Generate $\{\varphi_i := \varphi(\|\cdot - x_i\|), x_i \in X\}$



Imposing exactness of

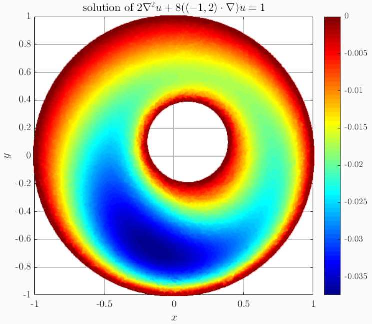
$$(\mathcal{L}u)(x_i) \approx \sum_{x_j \in N(x_i)} w_j^i u(x_j)$$

for each φ_j for $x_j \in N(x_i)$, we get

$$\begin{bmatrix} \varphi(\|x_{j_1} - x_{j_1}\|) & \cdots & \varphi(\|x_{j_{n_i}} - x_{j_1}\|) \\ \vdots & \ddots & \vdots \\ \varphi(\|x_{j_1} - x_{j_{n_i}}\|) & \cdots & \varphi(\|x_{j_{n_i}} - x_{j_{n_i}}\|) \end{bmatrix} \begin{bmatrix} w_{j_1}^i \\ \vdots \\ w_{j_{n_i}}^i \end{bmatrix} = \begin{bmatrix} (\mathcal{L}\varphi_{j_1})(x_i) \\ \vdots \\ (\mathcal{L}\varphi_{j_{n_i}})(x_i) \end{bmatrix}$$

Problem:

$$\mathcal{L}u = f \quad \text{on } \Omega,$$
$$u = u_0 \quad \text{on } \partial\Omega,$$



1. Discretize domain Ω
2. Find neighborhoods $N(x_i)$
3. Compute weights $\boldsymbol{w}^i$ for approximation of $\mathcal{L}$ over $N(x_i)$
4. Assemble weights in a sparse system $Wu = f$
5. Solve the sparse system $Wu = f$
6. Approximate/interpolate the solution

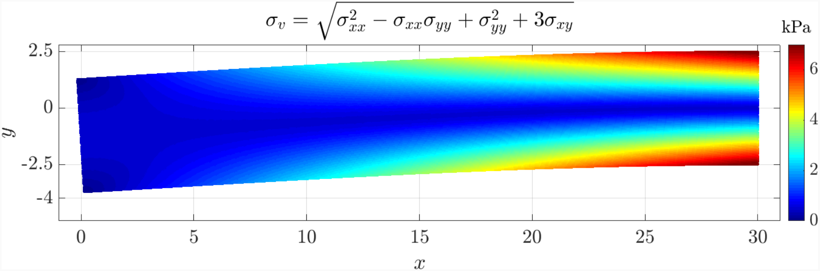
Cauchy-Navier equation

$$(\lambda + \mu)\nabla(\nabla \cdot \vec{u}) + \mu \nabla^2 \vec{u} = \vec{f}$$

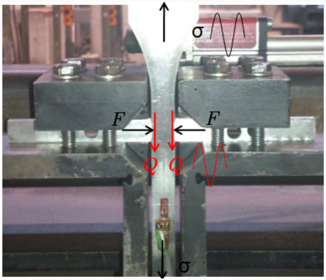
with stresses given as

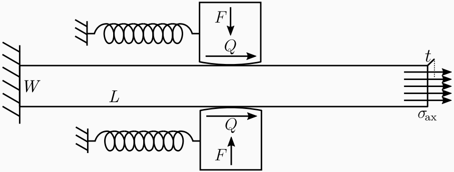
$$\sigma = \lambda \operatorname{tr}(\varepsilon) I + 2\mu\varepsilon, \quad \varepsilon = \frac{\nabla \vec{u} + (\nabla \vec{u})^\mathsf{T}}{2}.$$

Standard test case: cantilever beam



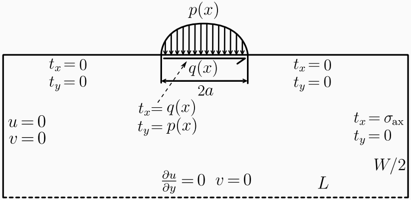
A thin specimen is axially stretched and compressed in another axis by two oscillating pads.



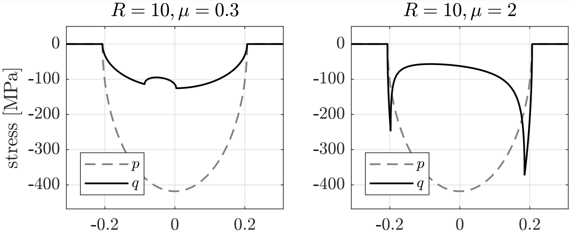


During simulation of a loading cycle, stresses need to be compute to apply material wear or initiate crack propagation.

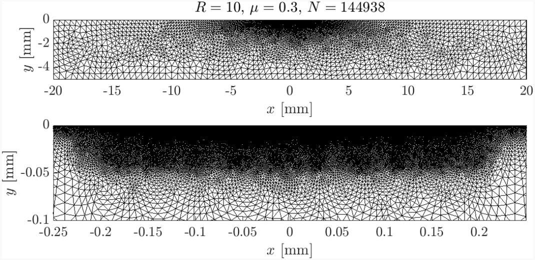
Taking into account the symmetry and imposing analytical BCs:

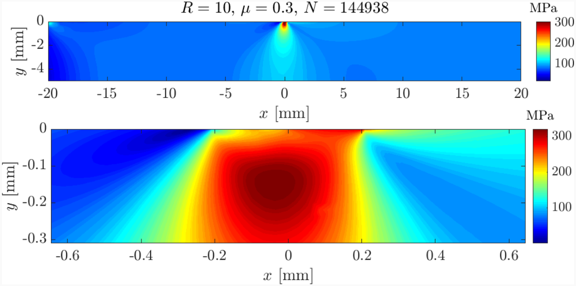


Top traction profile:



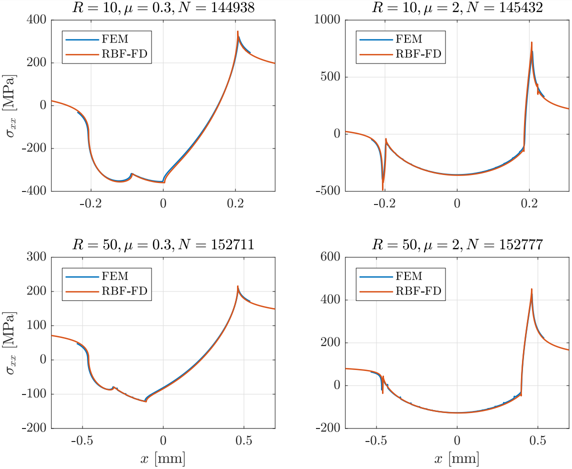
The contact area is 200 times smaller than domain width.



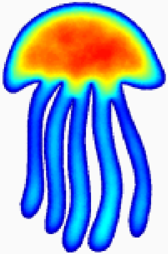


Fretting fatigue: numerical results

Comparison of stress profiles under contact:



All computations were done using open source Medusa library.



Medusa

Coordinate Free Mehless Method implementation

`http://e6.ijs.si/medusa/`

Thank you for your attention!

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